Components of AC circuit

Objectives

After going through this module learner will be able to

- Differentiate between the response of different components in AC and DC circuits
- Understand electrical circuits using pure resistor, inductor and capacitor
- Appreciate reactance offered by capacitor and inductor in ac circuits
- Represent current and voltage using phasors for pure R,L and C circuits

Content Outline

- Unit Syllabus
- Module Wise Distribution of Unit Syllabus
- Words You Must Know
- Introduction
- Components of AC Circuits
- Alternating Voltage Applied to a Resistor
- Representation of Alternating Current and Voltage by Graphs and Phasors
- Alternating Voltage Applied to an Inductor
- Alternating Voltage Applied to a Capacitor
- Summary

Unit Syllabus

Unit IV: Electromagnetic Induction and Alternating Currents

Chapter-6: Electromagnetic Induction

Electromagnetic induction; Faraday's laws, induced emf and current; Lenz's Law, Eddy currents; Self and mutual induction.

Chapter-7: Alternating Current

Alternating currents, peak and rms value of alternating current/voltage; reactance and impedance; LC oscillations (qualitative treatment only), LCR series circuit, resonance; power in AC circuits, wattless current; AC generator and transformer.

Module Wise Distribution of Unit Syllabus 09 Modules

The above unit is divided into 9 modules for better understanding.

Module 1	• Electromagnetic induction • Faraday's laws, induced emf and current; • Change of flux • Rate of change of flux
Module 2	• Lenz's Law, • Conservation of energy • Motional emf
Module 3	• Eddy currents. • Self induction • Mutual induction. • Unit • Numerical
Module 4	• AC generator • Alternating currents, • Representing ac • Formula • Graph • Phasor • Frequency of ac and what does it depend upon • Peak and rms value of alternating current/voltage;
Module 5	• ac circuits • Components in ac circuits • Comparison of circuit component in ac circuit with that if used in dc circuit • Reactance mathematically • Pure R • Pure L • Pure C • Phasor, graphs for each
Module 6	• AC circuits with RL, RC and LC components • Impedance; LC oscillations (qualitative treatment only), • Resonance • Quality factor

Module 7	● Alternating voltage applied to series LCR circuit ● Impedance in LCR circuit ● Phasor diagram ● Resonance ● Power in ac circuit ● Power factor ● Wattles current
Module 8	● Transformer
Module 9	● Advantages of ac over dc ● Distribution of electricity to your home

Module 5

Words You Must Know

Let us remember the words we have been using in our study of this physics course:

- **Electromagnetic Induction:** The phenomenon in which electric current can be generated by varying magnetic fields is called electromagnetic induction (EMI).
- **Magnetic Flux:** Just like electric flux, magnetic flux $\O_B$ through any surface of area A held perpendicularly in magnetic field B is given by the total number of magnetic lines of force crossing the area. Mathematically, it is equal to the dot product of B and A.

  $\Phi_B = B.A = BA\cos\theta$, where θ is the angle between B and A.
- **Induced emf and Induced current:** The emf developed in a loop when the magnetic flux linked with it changes with time is called induced emf when the conductor is in the form of a closed loop, the current induced in the loop is called an induced current.
- **Weber:** One weber is defined as the amount of magnetic flux, through an area of $1m^2$ held normal to a uniform magnetic field of one tesla. The SI unit of magnetic flux is weber (Wb) or tesla metre squared (Tm^2).

- **Faraday's Laws of Electromagnetic Induction:**
  - **First Law:** It states that whenever the amount of magnetic flux linked with the coil changes with time, an emf is induced in the coil. The induced emf lasts in the coil only as long as the change in the magnetic flux continues.
  - **Second Law:** It states that the magnitude of the emf induced in the coil is directly proportional to the time rate of change of the magnetic flux linked with the coil.

- **Lenz's Law:** The law states that the direction of induced emf is always such that it opposes the change in magnetic flux responsible for its production.
- **Fleming's Right Hand rule:** Fleming's right hand rule gives us the direction of induced emf/current in a conductor moving in a magnetic field.
- If we stretch the fore-finger, central finger and thumb of our right hand mutually perpendicular to each other such that the fore-finger is in the direction of the field, the thumb is in the direction of motion of the conductor, then the central finger would give the direction of the induced current.
- **Induced emf by Changing the Magnetic Field:** The movement of magnet or pressing the key of coil results in changing the magnetic field associated with the coil, this induces the emf.
- **Induced emf by Changing the Orientation of Coil and Magnetic Field:** When the coil rotates in a magnetic field the angle Ɵ changes and magnetic flux linked with the coil changes and this induces the emf. This is the basis of ac generators.
- **Induced emf by Changing the Area A: MOTIONAL EMF:** Motional emf is a type of induced emf which occurs when a wire is pulled through the magnetic field. The magnitude of motional emf depends upon the velocity of the wire, strength of magnetic field and the length of the wire.
- **Electric Current:** An electric current equals the rate of flow of electric charge. In electric circuits this charge is often carried by moving electrons in a wire. It can also be carried by ions in an electrolyte, or by both ions and electrons such as in plasma.
- **Voltage:** the difference in electric potential energy between two points per unit electric charge, in an electric circuit.
- **Ohm's Law:** Electric current through a conductor is directly proportional to the potential difference across the conductor provided the temperature and physical conditions of the conductor remains the same
- **Ohmic Conductors:** Conductors that follow ohm's law for reasonable range of physical conditions. conductor wires , conductor plates, strips
- **Non-Ohmic Conductors:** Conductors that do not follow Ohm's law example electrolytes, semiconductors
- **Eddy Currents:** Eddy currents are loops of electrical current induced within conductors by a changing magnetic field in the conductor, (as per Faraday's law of induction). Eddy currents flow in closed loops within conductors, in planes

perpendicular to the magnetic field. They can be induced within (nearby) stationary conductors by a time-varying magnetic field.

- **Phasors:** In physics and engineering, a phasor, is a complex number representing a sinusoidal function whose amplitude (A), angular frequency (ω), and initial phase (θ) are time-invariant. Basically, Phasors are rotating vectors.
- **Alternating Voltage:** The electric mains supply in our homes and work places is a voltage that varies like a function with time. Output from an ac generator.
- **Alternating Current:** Current in a circuit driven by ac voltage is called ac current.
- Alternating currents and voltages have **frequency f** and **angular frequency 2πf** associated with it.
- Two currents, two voltages or currents and voltages may have a phase relation between them. This arises due to electromagnetic induction, self induction or time rate associated with charging and discharging of capacitors.
- Alternating currents and voltages have **instantaneous value** given by:

  $i = i_0 \sin(\omega t + \Phi)$

  $V = V_0 \sin(\omega t + \Phi)$

  Φ is the initial phase of the sinusoidal current or voltage

  Alternating currents and voltages have peak value I_0 and V_0

  Alternating currents and voltages have average value over half cycle

  $V_{avg(T/2)} = \frac{2V_0}{\pi} \cong 0.636\, V_0$
- Alternating currents and voltages have root mean square values

  $V_{rms} = \frac{V_0}{\sqrt{2}}$
- **Self-Inductance of a Coil: L** An electric current can be induced in a coil by flux changes produced by the changing current in it self

  $L = \mu_0 n^2 A l$

  - Where nl = N total number of turns of the coil, A area of the face of the coil, μ_0 is the permeability of free space. Its SI unit henry.
  - Self-inductance is also called back emf. It depends upon the geometry of the coil and permeability of the medium inside the coil.
- **Energy Required to Build up Current I:** in a coil of inductance $L = \frac{1}{2} L I^2$.

- **Capacitor:** A system of two conductors separated by an insulator. Parallel plate capacitors, spherical capacitors are used in circuits. Capacity of parallel plate capacitor is given by
    - $c = \frac{\mu_0 A}{d}$. 'A' is the area of the plate, d separation between the plates, μ_0 is the permeability of free space.
    - Capacitors block dc but ac continues as charging and discharging of the capacitor maintains a continuous flow of current.
- **Capacitance:** $C = \frac{Q}{V}$ S.I. unit farad.
- **Dielectric Constant of a Material K:** Is the factor by which the capacitance increases from its vacuum value when the dielectric (material) is inserted fully between the plates of a capacitor.
- **Combination of Capacitors:** Capacitors may be combined in ways to obtain a value of effective capacitance.
- **Series combination:** Capacitances are said to be in series if the effective combined capacitance C is given by

  $$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots\ldots + \frac{1}{C_n}$$
- **Parallel Combination of Capacitors:** capacitances are said to be in series if the effective combined capacitance C is given by

  $$C = C_1 + C_2 + \ldots\ldots + C_n$$

Introduction

An electrical network is an interconnection of electrical components (e.g. batteries, resistors, inductors, capacitors, switches) or a model of such an interconnection, consisting of electrical elements (e.g. voltage sources, current sources, resistances, inductances, capacitances). An electrical circuit is a network consisting of a closed loop, giving a return path for the current.
AC electricity allows for the use of a resistor, capacitor and inductor within an electrical or electronic circuit. These devices can affect the way the alternating current passes through a circuit in a network. They are only effective with AC electricity.
Today, most of the electrical devices we use require ac voltage. This is mainly because most of the electrical energy sold by power companies is transmitted and distributed as alternating current. The main reason for preferring use of ac voltage over dc voltage is that ac voltages

can be easily and efficiently converted from one voltage to the other by means of transformers.

Further, electrical energy can also be transmitted economically over long distances. AC circuits exhibit characteristics which are exploited in many devices of daily use. For example –A combination of a capacitor, inductor and resistor is also used as a tuner in radios and televisions. These devices are very useful for tuning to different broadcasting stations.

Our mobiles connect to desired mobile numbers using electrical circuits that are able to choose the desired ten digit phone number.

We have studied electrical circuits that operate using cells and batteries. The output from a cell or a battery is direct current. We dealt with Ohm's law, Kirchhoff's law, Law of combination of resistors in series and parallel, circuit components like connecting wires, resistances, keys and current, voltage meters to determine current and voltages in the entire or branch of a circuit.

We have learnt about capacitors and inductors.

Now we will study electrical circuits powered not by cells and batteries but by alternating voltage sources.

Component of ac Circuits

In our study of DC electrical circuits, we dealt with resistances in the circuit which made the circuit operate according to required outcome. In AC circuits, capacitors and coils (inductors) are capable of changing the value of currents and voltages.

So, AC circuits will have alternating voltage source, resistances, capacitors and inductors switches and connecting wires.

Let us start with considering simple AC Circuits that contain only one circuit element (capacitor, or inductor or resistor), connected to an AC source.

Alternating Voltage Applied to a Resistor

A resistor is a passive two-terminal electrical component that provides a resistance as a circuit element. Resistors act to reduce current flow, and, at the same time, can be used to lower voltage levels within circuits. In electronic circuits, resistors are used to limit current flow, to adjust signal levels, bias active elements, terminate transmission lines, among other uses.

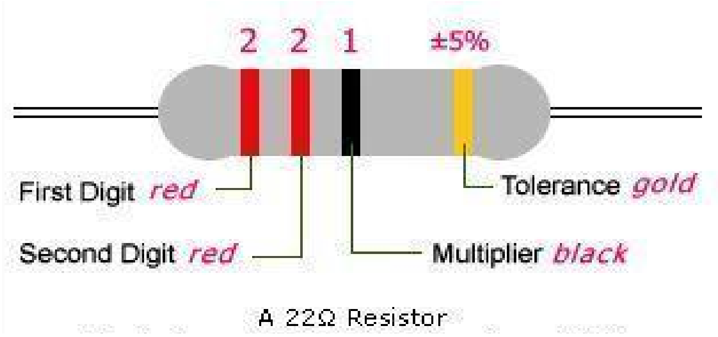
Many of these words are new to you but you will learn their meaning soon.

i. **High-power resistors** that can dissipate many watts of electrical power as heat.

ii. **Fixed resistors** have resistances that only change slightly with temperature, time or operating voltage.

iii. **Variable resistors** can be used as addictive circuit elements (such as used for audio volume control or a lamp dimmer, fan regulator), or as sensing devices for heat, light, humidity, force, or chemical activity.

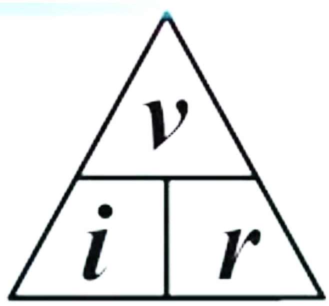
Symbolic carbon resistor



Source image: Wikipedia

You are familiar with Ohm's law connecting voltage, current and resistance

V=IR



The ohm (symbol: Ω) is the SI unit of electrical resistance, named after Georg Simon Ohm. An ohm is equivalent to a volt per ampere.

Since resistors are specified and manufactured over a very large range of values, the derived units of:

Milliohm (1 mΩ = 10^{-3} Ω),

Kilo-ohm (1 kΩ = 10^{3} Ω), and

Mega-ohm (1 MΩ = 10^{6} Ω) are also commonly used.

A resistive circuit is a circuit containing only resistors and ideal current and voltage sources.

Consider a purely resistive circuit with a resistor R connected to an AC generator, as shown in Figure given below.

The AC Source produces sinusoidally varying potential difference across its terminals.

The fig. shows a resistor connected to a source ε of ac voltage.

We consider a source which produces sinusoidally varying potential differences across its terminals. Let this potential difference, also called ac voltage, be given by.

$$V(t) = V_0 \, Sin \, \omega t$$

where V_0 is the amplitude of the oscillating potential difference and ω is its angular frequency.

To find the value of current through the resistor,

We apply Kirchhoff's Loop rule,

$\sum \varepsilon (t) = 0$ to the given purely resistive circuit

$$V_0 \, sin \, \omega \, t - I \, R \;=\; 0$$

$$I \;=\; \frac{V_0 \sin sin \, \omega t}{R}$$

Since R is a constant, we can write this equation as

$$I \;=\; I_0 \sin sin \, \omega t$$

Where the current peak value or maximum value will be given by

$$I_0 = \frac{V_0}{R}$$

Here I_0 is also called the current amplitude.

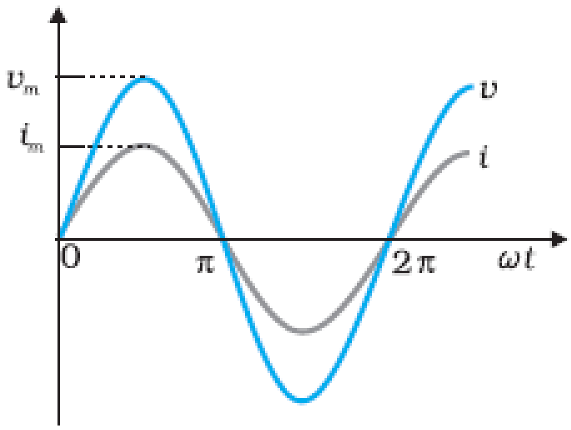
This shows that Ohm's Law for resistors is applicable to both ac and dc voltages.

Ohm's law holds for circuits containing only resistive elements (no capacitances or inductances) for all forms of driving voltage or current, regardless of whether the driving voltage or current is constant (DC) or time-varying such as AC.

At any instant of time Ohm's law is valid for such circuits.

Resistors which are in series or in parallel may be grouped together into a single "equivalent resistance" in order to apply Ohm's law for analyzing the circuit.

The time dependence of the current and the voltage across the resistor is depicted below:



In pure resistive circuit the voltage and current are in phase

From the graph, it is evident that both V and I reach zero, minimum (0) and maximum V_0 or V_m values at the same time.

Clearly, the voltage and current are IN PHASE with each other.

We also see that, like the applied voltage, the current varies sinusoidally and has corresponding positive and negative values during each cycle. **Thus, the sum of the instantaneous current values over one complete cycle is zero, and the average current is zero.**

The fact that the average current is zero, however, does not mean that the average power consumed is zero and **that there is no dissipation of electrical energy.**

As you know, Joule heating is given by **I^2R t** and depends on I^2 (which is always positive whether I is positive or negative) and not on instantaneous value of I.

Thus, there is Joule heating and dissipation of electrical energy when an alternating current passes through a resistor.

The instantaneous power dissipated in the resistor is:

$$p = I^2R = I_0^2 R sin^2 \omega t$$

The average value of *p* over a cycle is:

$$\bar{p} =< I^2R >=< I_0^2 R sin^2 \omega t >$$

$\bar{P}$ denotes its average value and <…..> denotes taking average of quantity inside the bracket

Since, I_0^2 $and\ R$ are constants

$$\bar{p} = I_0^2 R < sin^2 \omega t >$$

Using trigonometric identity $sin^2 \omega t = \frac{1}{2}(1 - cos 2\omega t)$

$$\{< sin^2 \omega t >\} \geq \frac{1}{2}$$

$$\bar{p} = I_0^2 R < sin^2 \omega t > \quad = \frac{1}{2} I_0^2 R$$

Or

$$\bar{p} = I_{rms}^2 R$$

$I_{rms} = \frac{I_0}{\sqrt{2}}$

This shows the advantage of introducing the concept of rms values. In terms of rms values, the equation for power and hence relation between current and voltage in ac circuits are essentially the same as those for the dc case.

It is customary to measure and specify rms values for ac quantities. For example, the household line voltage of 220 V is an rms value with a **peak voltage**

$V_0 = \sqrt{2}\,V_{rms}$

= (1.414) (220 V) = 311 V

In fact, the I_{rms} current is the equivalent dc current that would produce the same average power loss as the alternating current.

Note:

- **When a value is given for ac voltage or current, it is ordinarily the rms value. The voltage across the terminals of an outlet in your room is normally 240 V. This refers to the *rms* value of the voltage. The peak value of voltage or the amplitude of voltage is**

  $V_0 = \sqrt{2}V_{rms} = \sqrt{2}(240) = 340\,V$

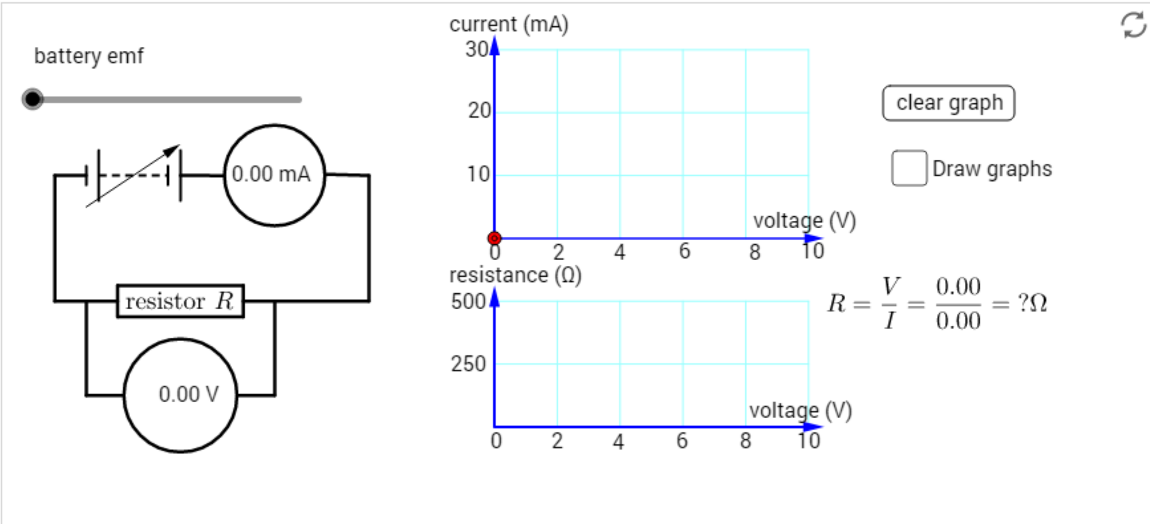
- **The power rating of an element used in ac circuits refers to its average power rating.**
- **The power consumed in a circuit is never negative.**
- **Both alternating current and direct current are measured in amperes.**

But how is the ampere defined for an alternating current? It cannot be derived from the mutual attraction of two parallel wires carrying ac currents, as the dc ampere is derived. An ac current changes direction with the source frequency and the attractive force would average to zero.

Thus, the ac ampere must be defined in terms of some property that is independent of the direction of the current. Joule heating is such a property, and there is one ampere of *rms* value of alternating current in a circuit if the current produces the same average heating effect as one ampere of dc current would produce under the same conditions.

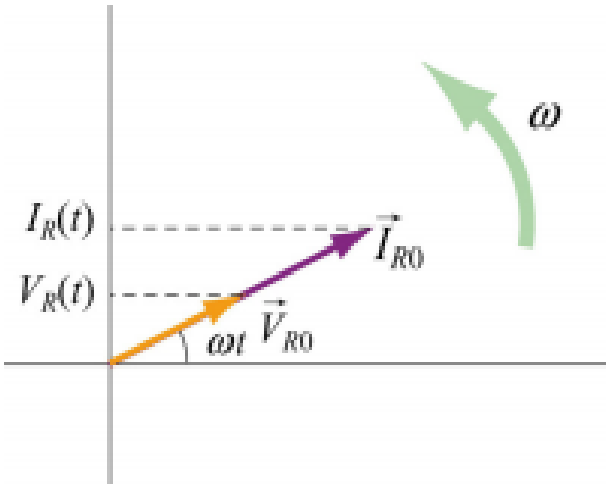
Representation of Alternating Current and Voltage by graphs and phasors

I-V characteristics (resistor)



Link here:

https://www.geogebra.org/m/K2gh8kfM?doneurl=%2Fsearch%2f
Perform%2Fsearch%2Fac%2b Circuit%2Baith%2Bresistor%2B%2Fmaterials%2F



The behavior of I_R (t) current through the resistor and V_R (t) voltage across the resistor, can also be represented with a phasor diagram.

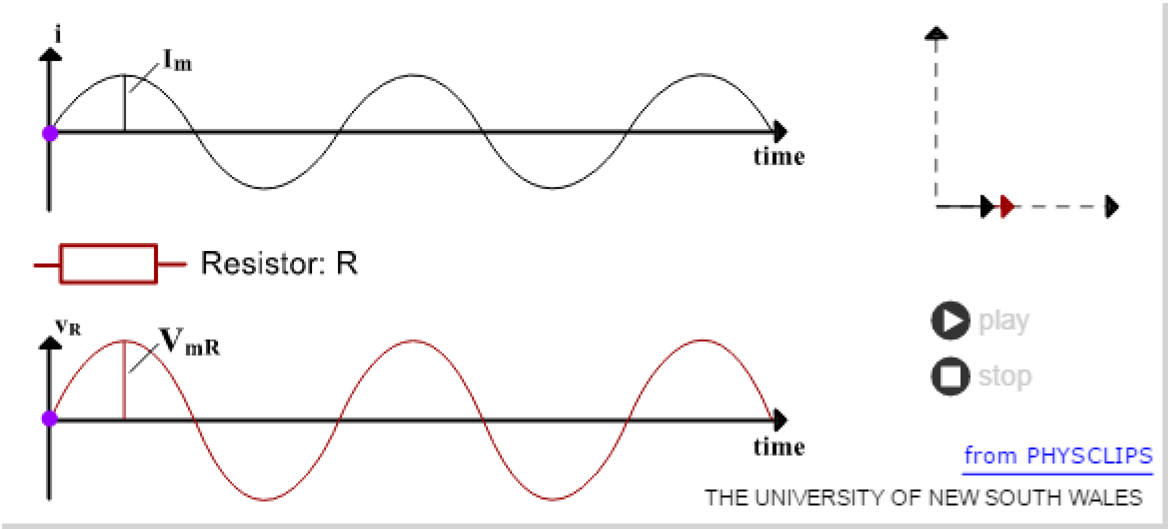
A phasor is a rotating vector having the following properties:

(i) Length: the length corresponds to the amplitude.

(ii) Angular speed: the vector rotates counterclockwise with an angular speed ω.

(iii) **Projection: the projection of the vector along the vertical axis corresponds to the value of the alternating quantity at time t.**

From the figure, we see that phasors V and I for the case of a resistor are in the same direction. This is so for all times. This means that the phase angle between the voltage and the current is zero.



Link here-

http://www.animations.physics.unsw.edu.au/jw/AC.html#resistors

This link allows you to imagine the current and voltage through a resistor circuit powered by an alternating voltage source.

Example: A light bulb is rated at 100W for a 220 V supply.

Find

a. The resistance of the bulb;
b. The peak voltage of the source; and
c. The rms current through the bulb.

Solution:

a. We are given $P = 100$ W and $V = 220$ V. The resistance of the bulb is

$$R = \frac{V^2}{P} = \frac{(200V)^2}{100W} = 484\ ohms$$

b. The peak voltage of the source

$$V_0 = \sqrt{2}\, V_{rms} = 311\ V$$

c. $P = I_{rms}\ V_{rms}$

$$I_{rms} = \frac{P}{V_{rms}} = \frac{100}{220} = 0.450A$$

Example:

A 100 Ω resistor is connected to a 220 V, 50 Hz ac supply.

a) What is the rms value of current in the circuit?

b) What is the net power consumed over a full cycle?

Solution:

a) $I_{rms} = \frac{V_{rms}}{R}$

$$I_{rms} = \frac{220}{100} = 2.20A$$

b) $P = I_{rms}^2 R = \frac{220 \times 220 \times 100}{100 \times 100} = 484\ W$

Example:

a. The peak voltage of an ac supply is 300 V. What is the rms voltage?

b. The rms value of current in an ac circuit is 10 A. What is the peak current?

c. Find the resistance in the circuit.

d. Draw a voltage time, current time graph, mark peak and rms values for each.

e. What is the phase relation between current and voltage?

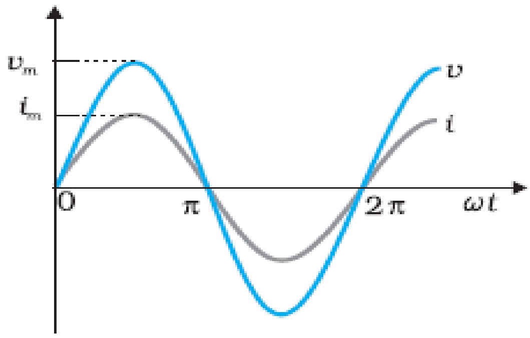
Solution:

a. $V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{300}{\sqrt{2}} = 0.707 \times 300 = 212.1\ V$

b. $I_0 = \sqrt{2} I_{rms} = 1.414 \times 10 = 14.14\ A$

c. $R = \frac{V_{rms}}{Irms} = \frac{V_0}{I_0} = \frac{300}{14.14} = 21.21\ ohms$

d.



rms current related to peak current.

Notice:

The peak voltage is 300 V and the peak current is 14.14 A, the y axis of the graph must have different scales to represent current and voltage.

e. The circuit has only a resistance hence current and voltage are in phase.

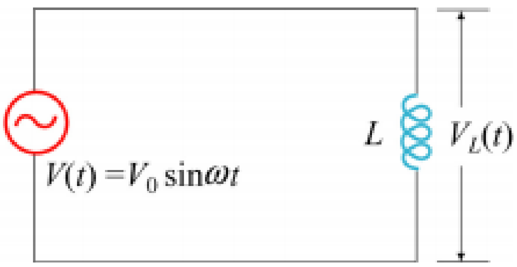
Alternating Voltage Applied to an Inductor

Now we are going to study a circuit which has a coil of negligible resistance and a source of alternating voltage. The reason to choose a negligible resistance coil is that if it were connected in a dc circuit infinite (very High) current would flow through it, but in alternating voltage circuit due to self inductance of the coil a "resistance" is offered to the current.

This is called **Inductive Reactance.** The coil is also called an inductor, the circuit is known as an **Inductive Circuit.**

So a coil may not offer resistance in a dc circuit because of low resistance, it offers reactance in an ac circuit. This is a very useful phenomenon.

Consider now a **purely inductive circuit**, with an inductor connected to an AC source, as shown below:



A purely inductive circuit

Usually, inductors have appreciable resistance in their windings, but we shall assume that this inductor has negligible resistance.

Thus, the circuit is a purely inductive ac circuit. Let the voltage across the source be

$V(t) = V_0 sin\omega t$

Using Kirchhoff's rule

$\sum e(t) = 0$

This implies,

$V(t) - L\frac{dI}{dt} = 0$

Notice:

- **The second term is the self induced Faraday emf in the coil or inductor and L is the self inductance of the coil**
- **The negative sign follows from Lenz's law**
- **V(t) implies Instantaneous voltage at instant t**
- **As voltage changes so does current in the circuit**

  $\frac{dI}{dt} = \frac{V(t)}{L} = \frac{V_0 sin\omega t}{L}$

The equation for *I*(*t*), the current as a function of time, must be such that its slope *dI/dt* is a sinusoidally varying quantity, with the same phase as the source voltage and an amplitude given by V_0/L.

To obtain the current, we integrate *dI/dt* with respect to time

$\int \frac{dI}{dt}dt = \frac{V_0}{L}\int sin\omega t\, dt$

We get

$I(t) = -\frac{V_0}{\omega L}cos\omega t + constant$

The integration constant has the dimension of current and is time independent.

Since the source has an emf which oscillates symmetrically about zero, the current it sustains also oscillates symmetrically about zero, so that no constant or time-independent component of the current exists.

Therefore, the integration constant is zero

Using

$$-\cos\theta = \sin\left(\theta - \frac{\pi}{2}\right)$$

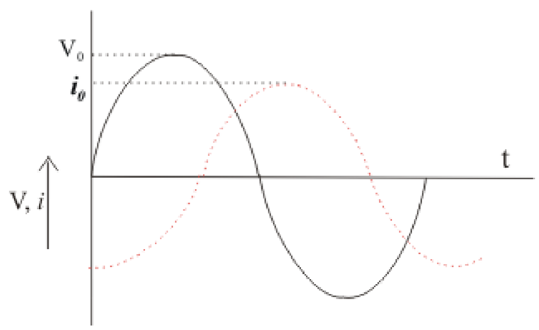
$$I(t) = \frac{V_0}{\omega L}\sin\left(\theta - \frac{\pi}{2}\right)$$

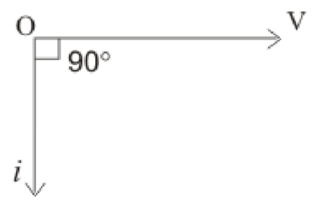
Here

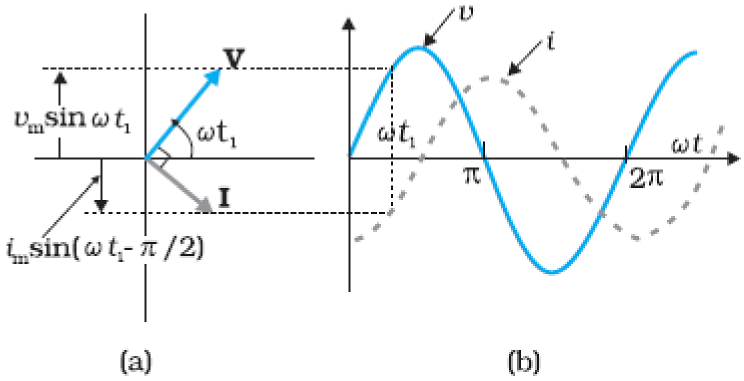
$\frac{V_0}{\omega L} = I_0$ is the peak value of current or amplitude of current

- The **quantity ωL is analogous to resistance** from ohm's law.
  It is represented by X_L
- Here, $X_L = \omega L = 2\pi f L$ is called the **inductive reactance**.

  The inductive reactance is directly proportional to the inductance and to the frequency of the current
- It has SI units of **ohms (Ω),** just like resistance.
- The inductive reactance limits the current in a purely inductive circuit in the same way as the resistance limits the current in a purely resistive circuit.
- However, unlike resistance, X_L depends linearly on the angular frequency ω. Resistance does not change with frequency. Thus the resistance to current flow increases with frequency.
- This is due to the fact that at higher frequencies the current changes more rapidly than it does at lower frequencies. Self inductance will have a larger value due to increased rate of change of flux.
- On the other hand, the inductive reactance vanishes as ω approaches zero.
- Also, the phase constant is: $\Phi = +\frac{\pi}{2}$
- The current and voltage plots and the corresponding phasor diagram for a purely inductive circuit are given below:







a) Time dependence of I_L (t) and V_L (t) across the inductor.

For the source voltage and the current in an inductor shows that the current lags the voltage by $\pi/2$ or one-quarter (1/4) cycle.

b) Phasor diagram for the inductive circuit.

As can be seen from the figures, the current I_L (t) is out of phase with V_L (t) by $\varphi = \pi / 2$; it reaches its maximum value after V_L (t) does by one quarter of a cycle.

Notice: The voltage and the current phasors in the present case at instant t_1. The current phasor I is $\pi/2$ behind the voltage phasor V. When rotated with frequency ω anticlockwise, they generate the voltage

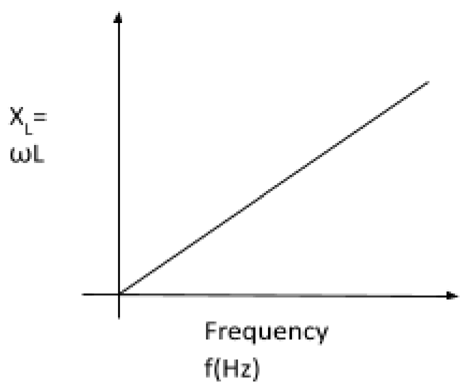
$$V(t) = V_0 sin\omega t$$

Current

$$I(t) = \frac{V_0}{\omega L} sin\left(\theta - \frac{\pi}{2}\right)$$

Thus Notice

- We say that the current lags voltage by $\pi / 2$ in a purely inductive circuit.
- We see that the current reaches its maximum value later than the voltage by one-fourth of a period $\left\{\frac{T}{4} = \frac{\frac{\pi}{2}}{\omega}\right\}$,
- Graph of X_L versus frequency of voltage



- Why would coils be called ‘choke coils’?
- Does an inductor consume power like a resistance?

The instantaneous power supplied to the inductor is

$$p = I(t)V(t) = I(t) = I_0 \, sin(\theta - \frac{\pi}{2}) \times V_0 sin\omega t$$

$$= - I_0 V_0 cos(\omega t) sin(\omega t)$$

$$= - \frac{I_0 V_0}{2} sin(2\omega t)$$

- Average power over one complete cycle = 0
- Does the coil store energy?
    - **As current increases in the coil, it sets up a magnetic field around it and energy is stored in the form of magnetic energy**
    - **Magnetic energy** $= \frac{1}{2} L I^2$
    - **Maximum value would correspond to maximum value of current**
    - **Maximum magnetic energy stored** $= \frac{1}{2} L I_0^2$

Think About This

a) On what factors does X_L depend upon?

b) A connecting wire of negligible resistance offers reactance in ac circuit. Why?

c) For the same coil of inductance L the reactance changes if the frequency is changed. Why?

d) What does the slope of the line indicate in the X_L- f graph?

e) How would you plot a graph such that the slope gives the value of self inductance?

f) Two coils of self inductance L_1 and L_2 are given to you. $L_1 > L_2$. Draw X_L- f for each if they are connected to the same frequency source. Which one of the two will have a greater slope? Why?

Example:

A pure inductor of 25.0 mH is connected to a source of 220 V. Find the inductive reactance and rms current in the circuit if the frequency of the source is 50 Hz.

Solution:

The inductive reactance,

$$X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 25 \times 10^{-3}$$

$$= 7.85 \; ohm$$

$$I = \frac{V}{X_L} = \frac{220}{7.8} = 28\,A$$

Try These:

i. **A 44 mH inductor is connected to 220 V, 50 Hz ac supply. Determine the rms value of the current in the circuit.**

ii. **Would your answer change?**

   a. **if frequency is 60 Hz**

   b. **If we use dc instead of ac**

   c. **If supply voltage is reduced to 110V**

   d. **If we consider the resistance of the inductor coil**

iii. **What is the phase relation between current and voltage?**

iv. **What will be the power lost in 2 min /1 hour?**

v. **In a pure inductive circuit the AC frequency is**

   i. **doubled**

   ii. **halved.**

vi. **How does the inductive reactance change in each case?**

Alternating Voltage Applied to a Capacitor

A capacitor (initially known as a condenser) is a passive two-terminal electrical component used to store electrical energy temporarily in an electric field.

The forms of practical capacitors vary widely, but all contain at least two electrical conductors (plates) separated by a dielectric (i.e. an insulator that can store energy by becoming polarized).

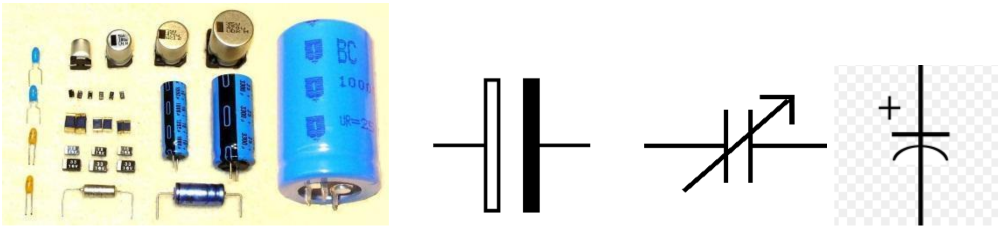
The conductors can be thin films, foils or sintered beads of metal or conductive electrolyte, etc. The non-conducting dielectric acts to increase the capacitor's charge capacity.

Materials commonly used as dielectrics include **glass, ceramic, plastic film, air, vacuum, paper, mica, and oxide layers.**

Capacitors are widely used as parts of electrical circuits in many common electrical devices.

Unlike a resistor, an ideal capacitor does not dissipate energy.

Instead, a capacitor stores energy in the form of an electrostatic field between its plates.



Source image: Wikipedia

In a DC circuit

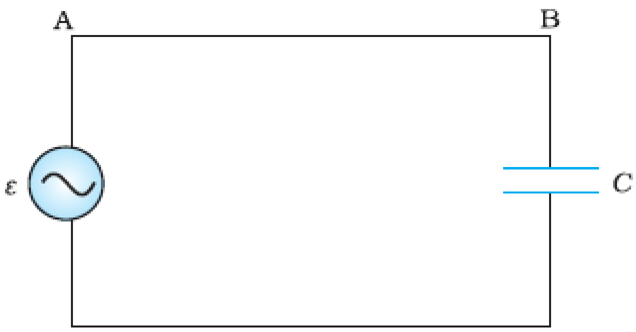
When a capacitor is connected to a voltage source in a dc circuit, current will flow for the short time required to charge the capacitor. As charge accumulates on the capacitor plates, the voltage across them increases, opposing the current. That is, a capacitor in a dc circuit will limit or oppose the current as it charges.

When the capacitor is fully charged, the current in the circuit falls to zero.

In an AC circuit

When the capacitor is connected to an ac source, as in, it limits or regulates the current, but does not completely prevent the flow of charge. The capacitor is alternately charged and discharged as the current reverses each half cycle.

The figure shows an ac source connected to a capacitor



Let q be the charge on the capacitor at any time t. The instantaneous voltage v is

$$V = \frac{q}{C}$$

If the supply alternating voltage is

$$V = V_0 sin\omega t$$

Using Kirchhoff's rule

$$V = V_0 sin\omega t = \frac{q}{C}$$

Current would be time rate of change of charge or $I = \frac{dq}{dt}$

$I = \frac{d}{dt}(V_0 C sin\omega t) = \omega C V_0 cos\, \omega t$

But $cos\, \omega t = sin(\omega t + \frac{\pi}{2})$

So we can write

$I = I_0 sin(\omega t + \frac{\pi}{2})$

Where

$I_0 = \omega C V_0 = \frac{V_0}{\frac{1}{\omega C}}$

Comparing it to $I_0 = \frac{V_0}{R}$ for a purely resistive circuit, we find that $(1/\omega C)$ plays the role of resistance. It is called **Capacitive Reactance** and is denoted by X_c,

- Here, $X_C = 1/\omega C$, is called the capacitance reactance.
- It also has SI units of ohms.
- Represents the effective resistance for a purely capacitive circuit.

  The capacitive reactance limits the amplitude of the current in a purely capacitive circuit in the same way as the resistance limits the current in a purely resistive circuit. But it is inversely proportional to the frequency and the capacitance.
- Note that X_C is inversely proportional to both C and ω,
- The phase constant is given by: $\Phi = -\frac{\pi}{2}$

Current and Voltage Phase Relationship in a Pure Capacitive ac Circuit

$V = V_0 sin\omega t$

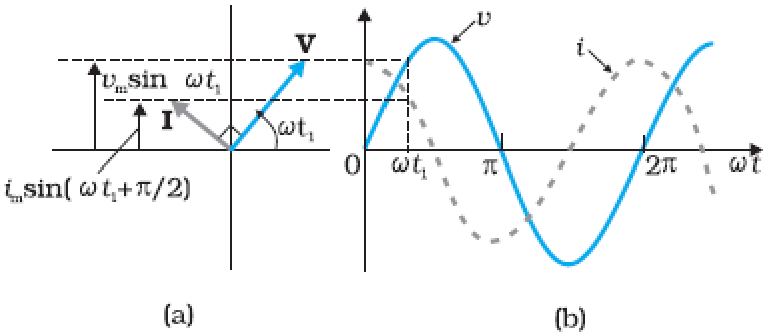
$I = I_0 sin(\omega t + \frac{\pi}{2})$

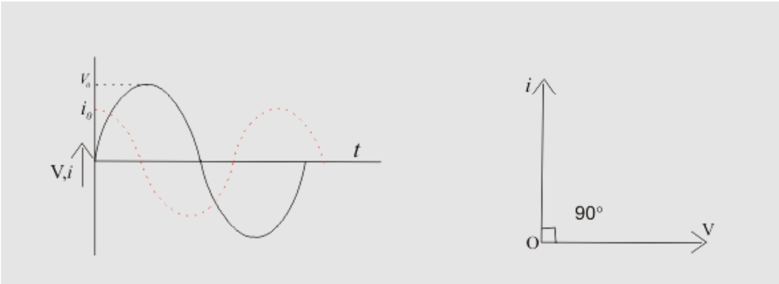
The current is π/2 ahead of voltage.

The graph shows the phasor diagram at an instant t_1. Here the current phasor I is π/2 ahead of the voltage phasor V as they rotate counterclockwise.

The graph shows the variation of voltage and current with time.

We see that the current reaches its maximum value earlier than the voltage by one-fourth of a period.





Notice that: at t = 0, the voltage across the capacitor is zero while the current in the circuit is at a maximum. Thus, we say that the current leads the voltage by π/2 in a capacitive circuit.

- **Does a capacitor consume power like a resistance?**

  The instantaneous power supplied to the capacitor is

  $p = I(t)V(t) = I(t) = I_0 \, sin(\theta - \frac{\pi}{2}) \times V_0 sin\omega t$

  $= - I_0 V_0 cos(\omega t) sin(\omega t)$

  $= - \frac{I_0 V_0}{2} sin(2\omega t)$

  So like in the case of an inductor the average power = 0

  As <sin(2ωt)> = 0 over a complete cycle

- **Does the capacitor store energy?**

  As charge increases in the capacitor, it sets up an electric field inside it and energy is stored in the form of electrostatic energy

  Electrostatic energy = $\frac{1}{2}CV^2$

  Maximum value would correspond to maximum value of voltage

  Maximum electrical energy stored = $\frac{1}{2}CV_0^2$

Thus, we see that

- **in the case of an inductor, the current lags the voltage by π/2**

- **In the case of a capacitor, the current leads the voltage by π/2.**

Example:

A lamp is connected in series with a capacitor. Predict your observations for dc and ac connections. What happens in each case if the capacitance of the capacitor is reduced?

Solution:

When a dc source is connected to a capacitor, the capacitor gets charged and after charging no current flows in the circuit and the lamp will not glow. There will be no change even if C is reduced. With ac source, the capacitor offers capacitive reactance ($1/\omega C$) and the current flows in the circuit. Consequently, the lamp will shine.

Reducing C will increase reactance and the lamp will shine less brightly than before.

Example:

A15.0 μF capacitor is connected to a 220 V, 50 Hz source.

Find the

a. capacitive reactance and
b. the current (rms and peak) in the circuit.

   If the frequency is doubled, what happens to the capacitive reactance and the current?

Solution:

a. The capacitive reactance is

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\times3.14\times50\times15\times10^{-6}} = 212\ ohms$$

b. The rms current is

$$I = \frac{V}{X_C} = \frac{220}{212} = 1.04\ A$$

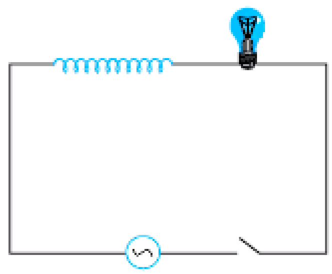
$$peak\ current\ will\ be\ I_0 = \sqrt{2}\ I = 1.41 \times 1.04 = 1.47\ A$$

This current oscillates between +1.47A and –1.47 A, and is ahead of the voltage by π/2.

If the frequency is doubled, the capacitive reactance is halved and consequently, the current is doubled.

Example:

A light bulb and an open coil inductor are connected to an ac source through a key as shown in Fig



The switch is closed and after some time, an iron rod is inserted into the interior of the inductor. The glow of the light bulb

a. increases;
b. decreases;
c. is unchanged,

As the iron rod is inserted. Give your answer with reasons.

Solution:

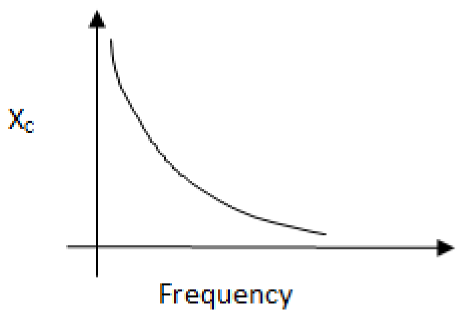
As the iron rod is inserted, the magnetic field inside the coil magnetizes the iron increasing the magnetic field inside it. Hence, the inductance of the coil increases. Consequently, the inductive reactance of the coil increases. As a result, a larger fraction of the applied ac voltage appears across the inductor, leaving less voltage across the bulb. Therefore, the glow of the light bulb decreases.

Example:

Plot a X_C – f graph

Solution:

$$X_C = \frac{1}{2\pi f C}$$



X_C inversely proportional to frequency if capacitance is constant

Problems for Practice

1. Consider a purely capacitive circuit (a capacitor connected to an AC source).
   a. How does the capacitive reactance change if the driving frequency is doubled? Halved?
   b. Are there any time instants when the capacitor is supplying power to the AC source?

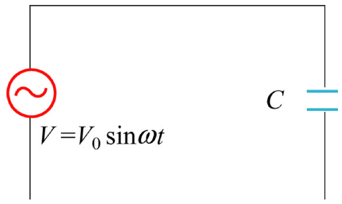
a. A capacitor of 0.5 μF capacitance, is connected, as shown in Figure (x) to an AC generator with V = 300 V. What is the amplitude I_0 of the resulting alternating current if the angular frequency ω is (i) 100 rad/s, and (ii) 1000 rad/s?

b. A 45-mH inductor is connected, as shown in Figure (y), to an AC generator with V = 300 V. The inductor has a reactance X_L = 1300Ω. What must be?

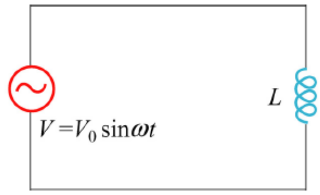
   i. The applied angular frequency ω and

   ii. The applied frequency f for this to be true?

   What is the amplitude I_0 of the resulting alternating current?



(x)



(y)

Figure (x) a purely capacitive circuit, and (y) a purely inductive circuit.

2. An AC voltage source is connected to a "black box" which contains a circuit, as shown in Figure x. Figure x A "black box" connected to an AC voltage source. The elements in the circuit and their arrangement, however, are unknown. Measurements outside the black box provide the following information:

$V(t) = (80\ V) sin\omega t$

$I(t) = (1.6\ A) sin\ (\omega t\ +\ 45^0)$

a. Does the current lead or lag the voltage?
b. Is the circuit in the black box largely capacitive or inductive?
c. Is the circuit in the black box at resonance?
d. What is the power factor?
e. Does the box contain a resistor? A capacitor? An inductor?
f. Compute the average power delivered to the black box by the AC Source.

3. A resistor of 200 Ω and a capacitor of 15.0 μF are connected in series to a 220 V, 50 Hz ac source. Calculate
   a. The current in the circuit;
   b. The voltage (rms) across the resistor and the capacitor.

   Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.

   a) 0.755 A; b) 220 V

4. At an airport, a person is made to walk through the doorway of a metal detector, for security reasons. If she/he is carrying anything made of metal, the metal detector emits a sound. On what principle does this detector work?

Solution:

The metal detector works on the principle of resonance in ac circuits. When you walk through a metal detector, you are, in fact, walking through a coil of many turns. The coil is connected to a capacitor tuned so that the circuit is in resonance. When you walk through with metal in your pocket, the impedance of the circuit changes, resulting in significant change in current in the circuit. This change in current is detected and the electronic circuitry causes a sound to be emitted as an alarm.

Suppose the frequency of the source in the previous example can be varied.

a. What is the frequency of the source at which resonance occurs?
b. Calculate the impedance, the current, and the power dissipated at the resonant condition.

   a) 35.4Hz; b) 3Ω, 66.7A, 13.35 kW

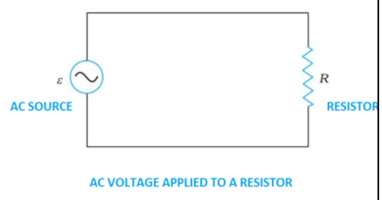
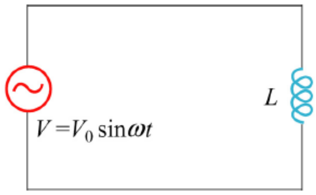
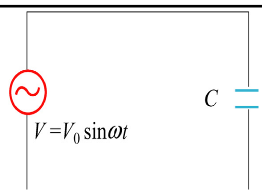
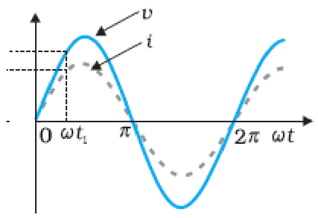
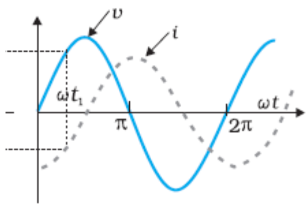
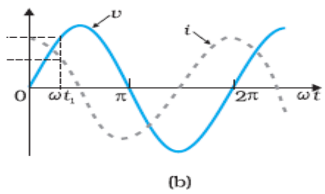
5. When an electric device x is connected to 220V AC supply, the current is 0.5 A and is in the same phase as potential difference, when another device y is connected to the same supply the current is again 0.5 A but it leads the potential difference by π/2 radians. Name the device x and y. (x resistance and y is a capacitor)
6. What is the meaning of wattles current?

Summary

In this module we have learnt:

- ac circuits
- components in ac circuits
- comparison of circuit component in ac circuit with that if used in dc circuit

- reactance mathematically
- ac circuit with pure R
- ac circuit with pure L
- ac circuit with Pure C
- Phasor, graphs for each

	Pure resistance circuit	Pure inductive circuit	Pure capacitive circuit
Circuit diagram			
Input voltage	**V**=$V_0 sin\omega t$	**V**=$V_0 sin\omega t$	**V**=$V_0 sin\omega t$
Current	**I**=$\frac{V_0}{R} sin\omega t$ **I**=$I_0 sin\omega t$	$I = I_0 sin(\omega t - \frac{\pi}{2})$	$I = I_0 sin(\omega t + \frac{\pi}{2})$
reactance	**R**	**X_L=2πfL**	**X_C=1/2πfC**
Current voltage graph			
Current voltage phasor